

# UNSTEADY HEAT CONDUCTION IN A QUARTER PLANE, WITH AN APPLICATION TO BUBBLE GROWTH MODELS

D. A. SPENCE and D. B. R. KENNING

Department of Engineering Science, Oxford University, Oxford, England

(Received 26 May 1977 and in revised form 21 October 1977)

**Abstract**—A solution is obtained to the problem of transient heat conduction in the quarter plane  $x, y \geq 0$ , initially at zero temperature, for the case of a suddenly-applied radiation condition on the boundary  $x = 0$ , the boundary  $y = 0$  being maintained at zero temperature. The solution is obtained in closed form by a double transform technique as an integral of the known solution for one-dimensional heat transfer in the half space  $x > 0$  subject to the radiation condition on  $x = 0$ . The temperature and heat flux on the boundaries have been found by quadrature.

The results are used to estimate the evaporation rate at the perimeter of a bubble growing (without a microlayer) on a plane wall of infinite conductivity in a uniformly superheated liquid. It is shown that evaporation at the wall may make a significant contribution to bubble growth at low Jacob number for fluids with large contact angle, or after the evaporation of a microlayer to dryness.

## NOMENCLATURE

- $a$ , bubble radius;  
 $a$ ,  $= \left( \frac{\pi}{12} \right)^{1/2} \frac{1}{Ja} \frac{ah}{k_l}$ ,  
 non-dimensional bubble radius;  
 $h$ , surface heat-transfer coefficient;  
 $Ja$ ,  $= k_l \Delta T / \lambda \rho_g \alpha_l$ , Jacob number;  
 $k$ , thermal conductivity;  
 $p$ , pressure, Laplace transform variable;  
 $q_1, q_2$ , local heat flux on  $x = 0, y = 0$  respectively;  
 $Q_1, Q_2$ ,  $= \int_0^\infty q_1 dy, \int_0^\infty q_2 dx$  respectively;  
 $R$ , gas constant;  
 $t'$ , time;  
 $t$ ,  $= \alpha \left( \frac{h^2}{k} \right) t'$ , non-dimensional time;  
 $T$ ,  $= U + V$ , temperature;  
 $\Delta T$ , superheat;  
 $U$ , temperature, one dimensional solution;  
 $\bar{U}$ , Laplace transform of  $U$ ;  
 $V$ , temperature;  
 $\bar{V}$ , Laplace transform of  $V$ ;  
 $V^*$ , Fourier sine transform of  $\bar{V}$ ;  
 $x, y$ , Cartesian coordinates made non-dimensional with respect to  $k/h$ ;  
 $z$ ,  $= t^{1/2}$ .

## Greek symbols

- $\alpha$ , thermal diffusivity;  
 $\lambda$ , latent heat;  
 $\xi$ , Fourier transform variable;  
 $\rho$ , density;  
 $\sigma$ , condensation coefficient.

## Subscripts

- $l$ , liquid;  
 $g$ , gas.

## 1. INTRODUCTION

THE PROBLEM of unsteady heat conduction in a quarter plane arises in the mathematical modelling of a number of physical situations. Analytical solutions are available from the known one-dimensional solutions when the boundary conditions are homogeneous [1] and have been given for the steady state in a region of specified length scale [1, 2], but a solution does not seem to have been published for an unsteady problem involving mixed boundary conditions.

In the present paper we consider one such problem, that in which one semi-infinite boundary of the quarter plane is isothermal and the heat flux on the other satisfies a radiation boundary condition. The study was promoted by an analysis of the growth of a vapour bubble on a wall in uniformly superheated liquid, Fig. 1. The heat flow to most of the curved surface of the bubble is given by one-dimensional transient conduction, but over a narrow band of width  $O[(\alpha_l t')^{1/2}]$

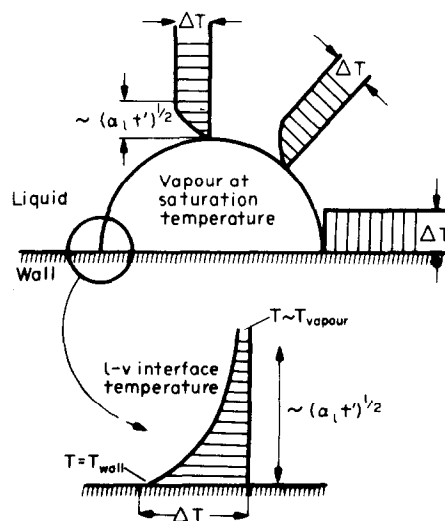


FIG. 1. Model for bubble growth on a wall in liquid at uniform superheat.

round the base of the bubble the heat flow is modified by the presence of the nearly-isothermal wall. In Section 3 we estimate this perturbation of the radial heat flow with the aid of the solution to the plane, two dimensional problem treated in Section 2. This solution has also been used in a discussion of transient thermocapillary flow [3]; it is relevant to other problems concerning transport and motion in the vicinity of a liquid-vapour-solid interface and to engineering problems such as thermal stress analysis at a junction between dissimilar materials suddenly subjected to cooling. It is also relevant to a number of geophysical problems: e.g. Kuznir and Bott [4] have studied the heat flow from a geothermal source by numerical methods using a model with similar mixed boundary conditions but a more complicated geometry.

In applications such as these, it is valuable to be able to compare the result of purely numerical computations with analytical solutions in a limiting case, and the present solution should be useful for this purpose. It would, for instance, be impossible numerically to elucidate the singularity at the corner, which must be correctly allowed for if a difference scheme is to be successful.

## 2. TRANSIENT CONDUCTION ANALYSIS

The problem we treat is that of two dimensional conduction in a quarter infinite solid,  $x, y \geq 0$ , initially at uniform temperature  $T = 0$ , which is suddenly brought into contact along its edge  $x = 0$  with a medium at different temperature  $T = 1$ , the heat-transfer coefficient on the boundary being  $h$ , while the temperature on the other edge  $y = 0$  is maintained throughout at the initial temperature.

If  $k/h$  is taken as the unit of length, and  $(k/h)^2/\alpha$  as the unit of time, the heat equation (with suffixes for partial derivatives) is

$$T_t = T_{xx} + T_{yy}. \quad (1)$$

The initial condition to be treated is

$$t = 0: T = 0 \quad x, y > 0 \quad (2)$$

and the boundary conditions are

$$x = 0: T - T_x = 1, \quad t > 0 \quad (3)$$

$$y = 0: T = 0, \quad t \geq 0. \quad (4)$$

The solution to this problem has been obtained by means of a Laplace transformation with respect to time followed by a Fourier sine transformation with respect to  $y$ , details of which are given in the Appendix. The final double transform for  $T(x, y, t)$  can be explicitly inverted, and here we simply quote the result. It is expressible in terms of the known solution  $U(x, t)$  of the one-dimensional heat equation in the half space  $x > 0$ , subject to the initial condition (2) and boundary condition (3), i.e. satisfying the equations

$$U_t = U_{xx}; \quad \left. \begin{aligned} U(x, 0) &= 0 & (x > 0) \\ U(0, t) - U_x(0, t) &= 1 & (t > 0) \end{aligned} \right\}. \quad (5)$$

When  $U(x, t)$  satisfies (5), then

$$T(x, y, t) = \int_0^t \operatorname{erf}\left(\frac{y}{2\tau^{1/2}}\right) U_x(x, \tau) d\tau \quad (6)$$

satisfies (1)–(4). This can be verified by direct substitution, after some partial differentiation and integration by parts, without use of the precise form of  $U$ ; and it follows from the analysis using transforms that the solution is unique. Carslaw and Jaeger ([1], p. 70) give

$$U(x, t) = \operatorname{erfc}\left(\frac{x}{2(t)^{1/2}}\right) - e^{x+t} \operatorname{erfc}\left(\frac{x}{2(t)^{1/2}}\right) + t^{1/2}. \quad (7)$$

The time derivative of this expression is

$$U_t(x, t) = (\pi t)^{-1/2} \exp\left(-\frac{x^2}{4t}\right) - e^{x+t} \operatorname{erfc}\left[\frac{x}{2(t)^{1/2}} + t^{1/2}\right]. \quad (8)$$

We are particularly interested in the temperature on the boundary  $x = 0$  and in the heat flux  $-q_2(x, t)$  say on the boundary  $y = 0$ . For computational purposes these are most easily calculated from their time-derivatives. These are found from (6) and (8):

$$(i) \quad \left(\frac{\partial T}{\partial t}\right)_{x=0} = [(\pi t)^{-1/2} - e^t \operatorname{erfc} t^{1/2}] \times \operatorname{erf}\left(\frac{y}{2t^{1/2}}\right) \quad (9)$$

$$(ii) \quad \frac{\partial q_2}{\partial t} = \left(\frac{\partial^2 T}{\partial y \partial t}\right)_{y=0} = (\pi t)^{-1/2} U_t(x, t). \quad (10)$$

When  $t = 0$ ,  $q_2 = 0$  for all  $x \geq 0$ . The heat transfer on the boundary is found by integrating (10) with respect to  $t$  for each  $x$ . The resulting integral is however singular at  $x = 0$ , where it behaves like  $-2/\pi \ln x$ .

When presenting the computed values on the boundary  $x = 0$  it is convenient to consider  $V$ , the deviation from the one-dimensional solution  $U$ , rather than  $T$ . We write

$$T(x, y, t) = U(x, t) + V(x, y, t). \quad (11)$$

Then

$$\left(\frac{\partial V}{\partial t}\right)_{x=0} = -[(\pi t)^{-1/2} - e^t \operatorname{erfc}(t^{1/2})] \operatorname{erfc}\left(\frac{y}{2t^{1/2}}\right). \quad (12)$$

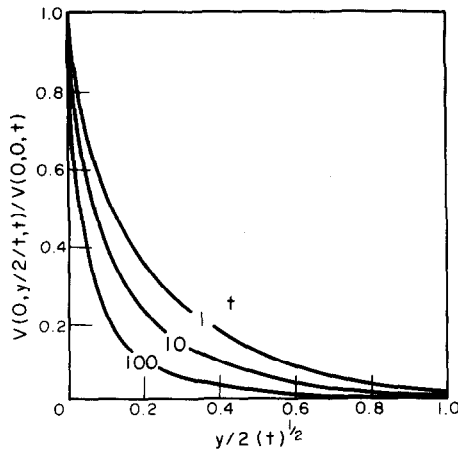
Equation (12) was integrated numerically using a Runge-Kutta sub-routine. The independent variable was taken as  $z = t^{1/2}$  rather than  $t$  itself. The range of values covered was  $0 \leq z \leq 10$  in steps of  $\Delta z = 0.1$ .

$V(0, y, t)$  is plotted as a function of  $t$  and  $y/2(t)^{1/2}$  in Fig. 2. This also represents the additional heat flux  $q_1$  say on  $x = 0$  due to the presence of the isothermal boundary  $y = 0$ , since

$$q_1(y, t) = (T_x - U_x)_{x=0} = (V_x)_{x=0}$$

by (2), (5), (11). The local additional heat flux is

$$Q_1 = \int_0^x q_1 dy.$$


 FIG. 2. Temperature and heat flux distribution on  $x = 0$ , normalised with respect to value at  $y = 0$ .

Its time derivative is given by (12) as

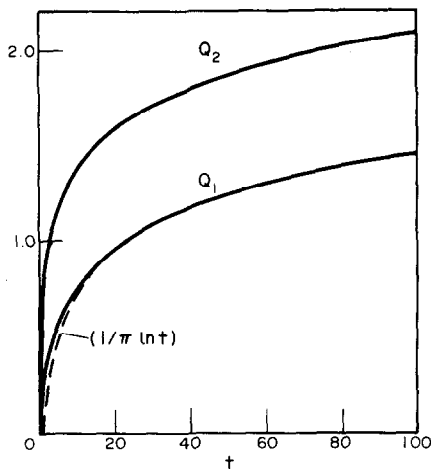
$$\frac{dQ_1}{dt} = \frac{2}{\pi} \{1 - (\pi t)^{1/2} e^t \operatorname{erfc} t^{1/2}\}. \quad (13)$$

For large  $t$ , this possesses the asymptotic expansion

$$\frac{\partial Q_1}{\partial t} \sim \frac{1}{\pi t} \left(1 - \frac{3}{2t} + \frac{15}{4t^2} - \dots\right). \quad (14)$$

For  $t > 20$ ,  $Q_1$  is within 1% of the asymptotic value  $(1/\pi) \ln t$ , Fig. 3, and the heat flow from  $t = 0$  is

$$\int_0^t Q dt \sim (t/\pi)(\ln t - 1).$$

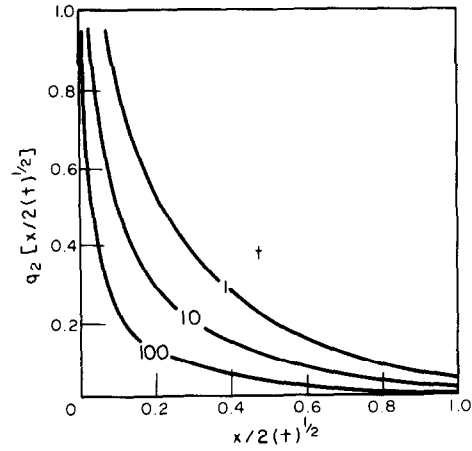

 FIG. 3. Heat flow  $Q_1$  across  $x = 0$ ,  $Q_2$  across  $y = 0$ .

On  $y = 0$  the temperature is zero and the local heat flux obtained by integrating equation (10) is given in Fig. 4. The total heat flux  $Q_2 = \int_0^\infty q_2 dx$  is given by

$$\frac{\partial Q_2}{\partial t} = \frac{e^t \operatorname{erfc}(t^{1/2})}{(\pi t)^{1/2}} \quad (15)$$

which

$$\sim \frac{1}{\pi t} \left(1 - \frac{1}{2t} + \frac{3}{4t^2} - \dots\right) \text{ for large } t. \quad (16)$$


 FIG. 4. Heat flux distribution on  $y = 0$ .

It is expected that  $Q_2 > Q_1$ , since the distortion of the isotherms near the origin reduces the internal energy of the medium, compared to the one-dimensional case. For  $t > 70$ ,  $Q_2 = Q_1 + 0.633$ .

### 3. BUBBLE GROWTH

A vapour bubble nucleated at a wall initially grows as a hemisphere. It obtains the heat required for evaporation (i) by conduction from the superheated liquid through a diffusion layer with thickness of order  $(\alpha_i t)^{1/2}$  and (ii) by conduction from the wall through the thin microlayer, formed at the base of the bubble by the action of viscosity at the wall. The microlayer dries out by evaporation at its centre and, for fluid-solid combinations with large contact-angle (e.g. water on many metals), may be rolled back towards the bubble perimeter by surface tension. While microlayers undoubtedly contribute to bubble growth under certain conditions, there are some discrepancies between experiment and theory which are discussed elsewhere (Mikić and Kenning, to be published). Here we seek to estimate the consequences for bubble growth of the total absence of a microlayer: its contribution to evaporation is lost, but instead there is intense evaporation at the triple liquid-vapour-solid interface, hitherto ignored in bubble growth calculations. This evaporation will be estimated from the preceding analysis by using the device of an effective heat-transfer coefficient to allow for the non-equilibrium which must occur between the liquid surface and the vapour at high rates of evaporation, estimated from kinetic theory to be [6],

$$h = \frac{2\sigma}{(2-\sigma)} \frac{\lambda}{(2\pi RT)^{1/2}} \left[ \left( \frac{dp}{dT} \right) - \frac{p}{2T} \right]. \quad (17)$$

Since we wish only to establish whether evaporation at the triple interface can make a significant contribution to bubble growth, we consider the idealised situation of hemispherical bubble growth in a uniformly superheated liquid with constant wall temperature. Conventionally the period of "asymptotic growth" is considered, when surface tension and liquid inertia may be neglected and the liquid-vapour interface far from the wall is close to the saturation

temperature. From the one-dimensional solution  $U(x, t)$ , this corresponds to  $t > 10^2$ ; typically bubble lifetimes lie in the range  $t = 10^3 - 10^6$ . Neglecting for the moment any influence of the wall, the heat flux from the bulk liquid to the hemispherical surface of the bubble is then given approximately by

$$q = \frac{k_l \Delta T}{(\pi \alpha_l t')^{1/2}} \quad (18)$$

More rigorous analysis in Lagrangian coordinates [7] leads to a correction factor of  $3^{1/2}$  to allow for the thinning of the diffusion layer by radial motion. The bubble growth equation is then

$$2\pi a^2 \cdot \lambda \rho_g \frac{da}{dt'} = 2\pi a^2 \cdot \frac{(3)^{1/2} k_l \Delta T}{(\pi \alpha_l t')^{1/2}} \quad (19)$$

or

$$\frac{da}{dt'} = Ja \left( \frac{3\alpha_l}{\pi t'} \right)^{1/2}, \quad Ja = \frac{k_l \Delta T}{\lambda \rho_g \alpha_l} \quad (20)$$

whence

$$a = Ja \left( \frac{12\alpha_l t'}{\pi} \right)^{1/2} \quad (21)$$

The Jacob number  $Ja$  can thus be interpreted as a measure of the ratio of the bubble radius  $a$  to the diffusion layer thickness  $(\alpha_l t')^{1/2}$  and a necessary condition for the applicability of equations (19)–(21) is that  $Ja \gg 1$ . We now make allowance for the influence of the wall, which affects the interface over a distance of order  $(\alpha_l t')^{1/2}$ . If  $Ja \gg 1$ , heat transfer over most of the hemispherical surface is unaltered and the additional heat flow at the triple interface can be simply added to the RHS of equation (19). We assume that the triple interface contribution is conservatively given by the asymptotic value for  $Q_1$ , derived in Section 2, without correction for radial motion or convection in the immediate vicinity of the contact line:

$$Q_1 = \frac{\ln t}{\pi}, \quad t > 20. \quad (22)$$

Equation (37) then becomes

$$2\pi a^2 \lambda \rho_g \frac{da}{dt'} = 2\pi a^2 \frac{(3)^{1/2} k_l \Delta T}{(\pi \alpha_l t')^{1/2}} + 2\pi a \frac{k_l \Delta T}{\pi} \ln \left[ \left( \frac{h}{k_l} \right)^2 \alpha_l t' \right] \quad (23)$$

or in non-dimensional variables

$$\frac{da^*}{dz} = 1 + \frac{z \ln z}{a^* 3Ja}, \quad z > 10 \quad (24)$$

where

$$a^* = \left( \frac{\pi}{12} \right)^{1/2} \frac{1}{Ja} \frac{ah}{k_l}, \quad z = t^{1/2}, \quad (25)$$

Equation (24) was integrated numerically with the arbitrary choice of initial condition  $a^* = t^{1/2}$  at  $t = 10^2$ , Fig. 5. It is seen that triple-interface evaporation makes a significant contribution to growth only for  $Ja \leq 15$ . By contrast, microlayer evaporation would increase  $a^*$  by a factor of approximately 2, the

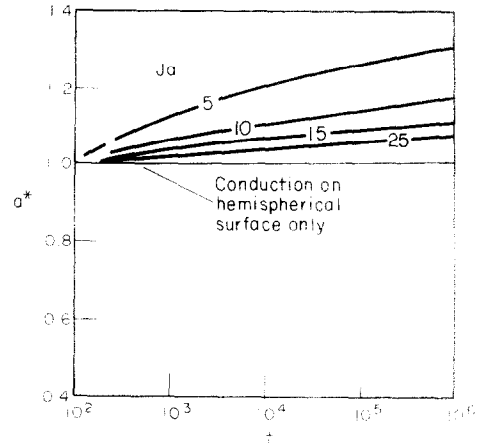


FIG. 5. Effect of evaporation at triple interface on bubble growth.

actual value depending on  $Ja$ ,  $\rho_l/\rho_g$  and the microlayer thickness [8]. In boiling, bubbles grow in a temperature gradient so that the contribution of the hemispherical surface to evaporation is reduced or goes negative, increasing the importance of wall evaporation. With increasing system pressure, Jacob numbers decrease and the potential evaporation at the triple interface may become comparable with that from a microlayer.

Because of the simplifying assumptions in the analysis, its application to boiling must be treated with caution. Convection in the immediate vicinity of a moving triple interface is not yet understood. A more important source of error may be the assumption that the wall is isothermal: the highly localised evaporation at the triple interface must cause temperature gradients in a wall of finite thermal diffusivity which will reduce the evaporation rate. Nevertheless, the analysis sounds a note of warning on the interpretation of experiments on bubble growth. Despite comments by Cooper and co-workers, e.g. [9, 10] on the possibility of microlayer roll-up, there is often an uncritical assumption that microlayer evaporation makes a significant contribution to growth. Since microlayers cannot be observed directly on metal surfaces, their presence is inferred from wall temperature measurements with microscopic thermometers and/or the observed bubble growth rate. The passage of the triple interface over a thermometer could equally well cause a variation in temperature (which would have to be interpreted by two-dimensional, rather than one-dimensional, conduction analysis) and its possible effect on bubble growth rate has already been noted.

*Acknowledgement*—Dr. J. R. Turner performed the numerical calculations for Section 2 of the paper.

#### REFERENCES

1. H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd Edn. Oxford University Press, Oxford (1959).
2. W. Karush, A steady-state heat flow problem for a quarter infinite solid, *J. Appl. Phys.* **23**, 492 (1952).
3. D. B. R. Kenning and H. Toral, On the assessment of thermocapillary effects in nucleate boiling of pure fluids, *Int. Conf. Phys. Chemistry and Hydrodynamics*, Oxford (1977).

4. N. J. Kuznir and M. H. P. Bott, A thermal study of the formation of oceanic crust, *Geophys. J. R. Astr. Soc.* **47**, 83 (1976).
5. A. Erdelyi, *Tables of Integral Transforms*, Vol. 1. McGraw-Hill, New York (1954).
6. R. W. Schrage, *A Theoretical Study of Interphase Mass Transfer*. Columbia University Press (1953).
7. M. S. Plesset and S. A. Zwick, The growth of vapour bubbles in superheated liquids, *J. Appl. Phys.* **25**, 493 (1954).
8. M. G. Cooper and J. M. D. Merry, A general expression for the rate of evaporation of a layer of liquid on a solid body, *Int. J. Heat Mass Transfer* **16**, 1811 (1973).
9. M. G. Cooper and A. J. P. Lloyd, The microlayer in nucleate pool boiling, *Int. J. Heat Mass Transfer* **12**, 895 (1969).
10. M. G. Cooper and J. M. D. Merry, Microlayer evaporation in nucleate boiling, Report CUEDA/A-Thermo/TRI, Engineering Department, Cambridge University (1970).

## APPENDIX

*Solution by double transform*

After writing  $T(x, y, t) = U(x, t) + V(x, y, t)$  (equation 11), we form the Laplace transform

$$\mathcal{L}V = \int_0^\infty e^{-pt} V(x, y, t) dt \equiv \bar{V}(x, y, p). \quad (A1)$$

This satisfies

$$\bar{V}_{xx} + \bar{V}_{yy} = p\bar{V} \quad (A2)$$

with boundary conditions

$$x = 0: V - \bar{V}_x = 0 \quad (A3)$$

$$y = 0: \bar{V} = -\bar{U} = -\frac{e^{-x}(p)^{1/2}}{p[1+(p)^{1/2}]} \quad (A4)$$

(A2) can be converted into an ordinary differential equation for the Fourier sine transform

$$\mathcal{F}\bar{V} = \int_0^\infty (\sin \xi y) \bar{V}(x, y, p) dy \equiv V^*(x, \xi, p). \quad (A5)$$

Application of the operator  $\mathcal{F}$  to (A2), (A4) gives

$$\frac{d^2 V^*}{dx^2} - (\xi^2 + p)V^* = \xi \bar{U} \quad (A6)$$

and (A3) becomes

$$x = 0: V^* = \frac{dV^*}{dx} = 0$$

whence

$$V^* = \frac{1}{\xi p} W^* - \frac{U(x, p)}{\xi}, \quad W^* = \frac{e^{-x(\xi^2 + p)^{1/2}}}{1 + (\xi^2 + p)^{1/2}}. \quad (A7)$$

The inverse Fourier transform of  $V^*$  is

$$\bar{V}(x, y, p) = \mathcal{F}^{-1}(W^*/\xi p) - \bar{U}(x, p) \quad (A8)$$

and Laplace inversion then gives

$$T(x, y, t) = \mathcal{L}^{-1}(\bar{V} + \bar{U}) = \mathcal{L}^{-1}\mathcal{F}^{-1}(W^*/\xi p). \quad (A9)$$

The order of the inverse operations can be reversed. The function  $W^*$  possesses a known inverse: from {[5], p. 246 (12)} we find

$$\mathcal{L}^{-1}W^* = e^{-\xi^2 t} U_i(x, t) \quad (A10)$$

where  $U_i$  is the expression quoted in (10).  $\mathcal{L}^{-1}(W^*/p)$  is the integral of this expression with respect to time.

Also

$$\begin{aligned} \mathcal{F}^{-1}(\xi^{-1} e^{-\xi^2 t}) &= \frac{2}{\pi} \int_0^\infty e^{-\xi^2 t} (\sin \xi y) \frac{d\xi}{\xi} \\ &= \operatorname{erf} \frac{y}{2(t)^{1/2}} \end{aligned} \quad (A11)$$

by {[5], p. 73 (21)}.

The full solution to our problem is therefore

$$T(x, y, t) = \int_0^t \left[ \operatorname{erf} \frac{y}{2(\tau)^{1/2}} \right] U_i(x, \tau) d\tau. \quad (A12)$$

## CONDUCTION THERMIQUE INSTATIONNAIRE DANS UN QUADRANT AVEC APPLICATION A DES MODELE DE CROISSANCE DE BULLE

**Résumé**—On donne une solution du problème de la conduction thermique variable dans le quadrant  $x, y > 0$ , initialement à température nulle, avec une condition de rayonnement sur la frontière  $x = 0$ , la frontière étant maintenue à température nulle. La solution est obtenue analytiquement, par une technique de double transformation, comme une intégrale de la solution connue pour le transfert monodimensionnel dans le demi-espace  $x > 0$  soumis à une condition de rayonnement sur  $x = 0$ . La température et le flux thermique sur les frontières ont été trouvés par quadrature. Les résultats sont utilisés pour estimer l'évaporation sur le périmètre d'une bulle en croissance (sans microcouche) sur une paroi plane de conductivité infinie dans un liquide uniformément surchauffé. On montre que l'évaporation à la paroi peut apporter une contribution sensible à la croissance de la bulle pour un faible nombre de Jakob avec un grand angle de contact, ou après l'évaporation d'une microcouche à l'assèchement.

## INSTATIONÄRE WÄRMELEITUNG IN EINEM FLÄCHENQUADRANTEN MIT ANWENDUNG AUF BLASENWACHSTUMSMODELLE

**Zusammenfassung**—Es wurde eine Lösung für das Problem instationäre Wärmeleitung im Flächenquadranten  $x, y \geq 0$  für den Fall eines plötzlich auftretenden Strahlungszustandes am Rand  $x = 0$  erhalten, wobei die Temperatur an der Begrenzung  $y = 0$  auf Null gehalten wurde. Die Lösung wurde in geschlossener Form durch eine Doppeltransformation als ein Integral der bekannten Lösung für den eindimensionalen Wärmeübergang in der Flächenhälfte  $x > 0$  erhalten, abhängig von den Strahlungsbedingungen bei  $x = 0$ . Die Temperatur und der Wärmestrom an den Rändern wurden durch Quadratur gefunden. Die Ergebnisse werden angewandt, um die Verdampfungsrate am Umfang einer wachsenden Blase (ohne Mikroschicht) an einer flachen Wand mit unendlicher Wärmeleitung in einer gleichmäßig überhitzten Flüssigkeit abzuschätzen. Es wird bestätigt, daß die Verdampfung an der Wand einen bedeutsamen Anteil am Blasenwachstum bei niedrigen Jacobs-Zahlen für Fluide mit großen Randwinkeln oder nach dem Verdampfen einer Mikroschicht bis zum Austrocknen hat.

ЗАДАЧА НЕСТАЦИОНАРНОЙ ТЕПЛОПРОВОДНОСТИ ДЛЯ КВАДРАТНОЙ  
ПЛАСТИНЫ ПРИМЕНИТЕЛЬНО К МОДЕЛЯМ РОСТА ПУЗЫРЬКОВ

**Аннотация** — Получено решение задачи нестационарной передачи тепла теплопроводностью в квадратной пластине,  $x, y \in [0, 1]$ , с начальной нулевой температурой для случая, когда граница  $x = 0$  внезапно испытывает действие излучения, а граница  $y = 0$  поддерживается при нулевой температуре. Решение получено в замкнутом виде методом двойного преобразования как интеграл известного решения для одномерного переноса тепла в полупространстве  $x > 0$  при воздействии излучения на границе  $x = 0$ . Температура и тепловой поток на границах получены в квадратурах. Результаты использовались для расчета скорости испарения по периметру пузырька, растущему (без микрослоя) на плоской стенке бесконечной проводимости в однородно перегретой жидкости. Показано, что испарение на стенке может оказывать значительное влияние на рост пузырьков при низком числе Якоба в жидкостях с большим углом смачивания или после полного испарения микрослоя.